1st SEM
Paper-2

LESSON PLAN

Class : 1st sem Sub : phy(hons) No. of Periods / Weeks :

Sl No.	Month	Paper/ Unit	Topics assigned	Page No.
1	2	3	4	5
	January	P-2 Unit II	Elasticity Relⁿ betⁿ elastic constant, twisting torque on a cylinder or wire. Bending of beams, External bending moment, flexural rigidity single and double cantilever. Fluid motion kinematics of moving fluids, Poiseuille's eqⁿ through a capillary tube. surface tension, Gravitational waves ripple Viscosity! - Poiseuille's eqⁿ for flow of liquid with corrections	

PROGRESS

Sl No	Date	Time	Topics covered (If class not taken, mantion the reasons)	Signature of Teacher
1	2	3	4	5
	3.1.21	10-11	- Introduction for selfstudy	[illegible]
	5.1.21	-do-	Brief discussion about - twisting torque on a cylinder or wire - Bending of Beams flexural rigidity	[illegible]
	7.1.21	-do-	Revision & Discussion - kinematics of moving fluids - Poiseuille's eqn through capillary tube - Gravitational waves & ripples - ~~[illegible]~~	[illegible]
	9.1.21	do	- Revision viscosity	[illegible]
	10.1.21	-do-	Doubt clearing with some question.	

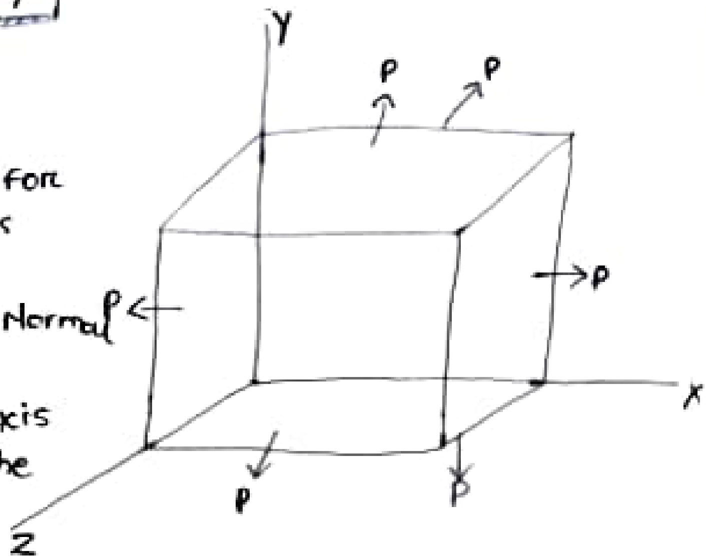
UNIT-2

CHAPTER-3

ELASTICITY

Relation Between Elastic Constant :-

Relation Betⁿ Y, k & σ :-



- Let us consider a cube having length 'L' for each side which is parallel to the co-ordinates x, Y & z.
- The cube is subjected to a uniform Normal P for each side of face.

The stress 'P' acting parallel to X axis will produce extension parallel to the x-axis but compression parallel to y and z axis. Same for Y & z axis.

The linear strain along x axis due to tensile stress along x axis is
But lateral strain along X axis due to tensile stress along Y a
is $-\frac{\sigma P}{Y}$.

And the strain produce at x axis due to stress alon z axis is $-\frac{\sigma P}{Y}$

imilarly considering other two pair of stress along Y & z axis
he resultant strain along X or y or z is

$$\frac{P}{Y} - \frac{\sigma P}{Y} - \frac{\sigma P}{Y} = \frac{P}{Y}(1-2\sigma) \quad \text{——— (1)}$$

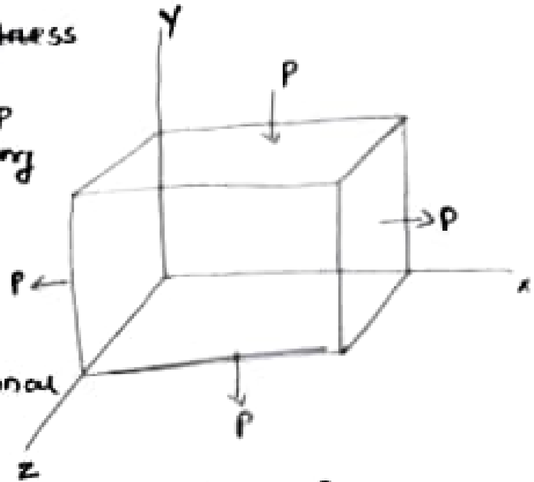
$$\text{volume strain} = \frac{3P}{Y}(1-2\sigma) \quad \text{——— (2)}$$

$$\text{Bulk modulus } (k) = \frac{P}{\text{volume strain}} \Rightarrow k = \frac{P}{\frac{3P}{Y}(1-2\sigma)}$$

$$\boxed{k = \frac{Y}{3(1-2\sigma)}} \quad \text{——— (3)}$$

~~Consider a cube subjected to tensile stress~~

Consider a cube having external stress P along x-axis and compressional stress P along Y axis. But no stress along z-axis.



The linear strain along x-axis due to stress along x-axis is $\frac{P}{Y}$

And strain along x-axis due to compressional stress along Y-axis is $\frac{\sigma P}{Y}$

The resultant strain along x-axis is $\frac{P}{Y} + \frac{\sigma P}{Y} = \frac{P}{Y}(1+\sigma)$ — (1)

Similarly resultant strain along Y and z axis are

$\frac{P}{Y} + \frac{\sigma P}{Y}$ & $-\frac{\sigma P}{Y} + \frac{\sigma P}{Y} = 0$ respectively

~~simultaneously equal~~ Both extensional & compressional strains are right angle to each other is equal to shear strain (θ)

$$\therefore \theta = \frac{P}{Y}(1+\sigma) + \frac{P}{Y}(1+\sigma) \quad \text{— (2)}$$

$$\cancel{\frac{P}{\theta}} \Rightarrow \frac{P}{\theta} = \frac{Y}{2(1+\sigma)}$$

(But $\frac{P}{\theta} = \eta$)

$$\boxed{\eta = \frac{Y}{2(1+\sigma)}} \quad \text{— (3)}$$

where η is modulus of Rigidity

Relation bet^n Y, K and η :-

We know $K = \frac{Y}{3(1-2\sigma)}$

$$\Rightarrow 1 - 2\sigma = \frac{Y}{3K} \quad \text{— (1)}$$

And also we know $\eta = \frac{Y}{2+2\sigma}$

$$\Rightarrow 2 + 2\sigma = \frac{Y}{\eta} \quad \text{— (2)}$$

Adding eq^n (1) & (2) we get.

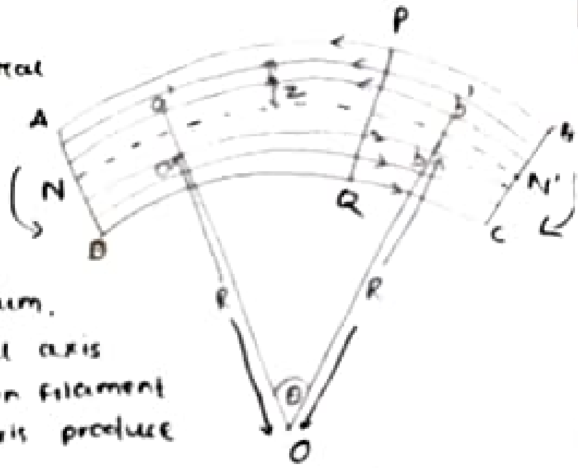
$$\Rightarrow 1 - 2\sigma + 2 + 2\sigma = \frac{Y}{3K} + \frac{Y}{\eta}$$

$$\Rightarrow 3 = Y\left(\frac{1}{3K} + \frac{1}{\eta}\right)$$

$$\Rightarrow \frac{3}{Y} = \frac{1}{3K} + \frac{1}{\eta}$$

$$\Rightarrow \boxed{\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}} \quad \text{— (3).}$$

External Bending Moment:-



- In the figure NN' represent as natural axis the filament above NN' are elongated and are at tension while the filament below the axis are compresed.
- PQ divides the beam in to two part take the PQBC which at equilibrium.
- The filment of above the natural axis produce external force in-ward on filament while the filament below the axis produce compressional force out-ward
- The in ward & outward fore constitute a couple called restoring f which act anti clockwise
- in portion ~~PQBC~~ PDCQ is in eqm state & the restoring force acts in clockwise.
- To finding moment of restoring force we consider a small portion of bent beam of angle ϕ at center of curvature O, having radius R.
- consider a filament a'b' at a distance z from NN'.

$$a'b' = (R+z)\phi$$
$$ab = R\phi$$

change in length is

$$\Rightarrow a'b' - ab = (R+z)\phi - R\phi = z\phi \quad \text{—— ①}$$

strain produce is $\dfrac{\text{change in length}}{\text{initial length}} = \dfrac{z\phi}{R\phi} = \dfrac{z}{R}$ —— ②

let f be the force acting on cross-section & δa be its area of cross-section then

$$\text{Young modulus } Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{f/\delta a}{z/R}$$

$$\Rightarrow \frac{f}{\delta a} = \frac{Yz}{R}$$

$$f = \frac{Y(\delta a)z}{R} \quad \text{—— (3)}$$

moment of this force along natural axis is

$$fz = \frac{Y(\delta a)z}{R}\cdot z$$
$$= \frac{Yz^2(\delta a)}{R} \quad \text{—— (4)}$$

Sum of all moment of force acting on crossection is

$$\Sigma fz = \Sigma \frac{Yz^2(\delta a)}{R} = \frac{Y}{R}\Sigma(\delta a)z^2 \quad \text{—— 5}$$

$\Sigma(\delta a)z^2$ is known as geometrical moment of inertia (I_g)

$$\boxed{\text{Bending Moment} = \frac{YI_g}{R}} \quad \text{—— (7)}$$

...al Rigidity:-

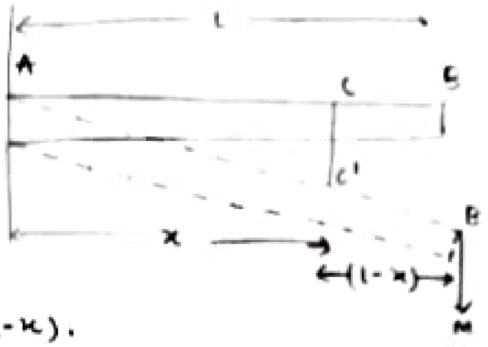
Flexural Rigidity :-

The quantity YI_g is known as Flexural Rigidity.

It is defined as " the bending moment required to produce unit radius of curvature in the beam". [for rectangle section $I_g = \frac{bd^3}{12}$, " circular section $I_g = \frac{\pi r^4}{4}$]

CANTILEVER :- (Light cantilever)

(1) Let us consider a thin cantilever AB of length L which is fixed at one end A & B is loaded with weight W as shown in figure.



Due to load B goes downward and form the dotted line.

Consider a section CB at x distance from A. W produce a load on CB at clockwise. The magnit of torque is $W(L-x)$.

~~The torque is formed by internal force due elasticity inside the beam. it act on CB is given by~~

The torque on section CB due to elasticity of beam is $\frac{YI_g}{R}$.

The two forces must balance each other

$$W(L-x) = \frac{YI_g}{R} \quad \text{—— (1)}$$

A is origin & the co-ordinates of C is (x, y) then radius of curvature is

$$\frac{1}{R} = \frac{d^2y/dx^2}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}} \quad \text{—— (2)}$$

But $\frac{dy}{dx} << \left(\frac{dy}{dx}\right)^2$ so eq^n (2) become $\frac{1}{R} = \frac{d^2y}{dx^2}$ —— (3)

$\therefore$ eq^n (1) become $W(L-x) = YI_g \frac{d^2y}{dx^2}$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{W}{YI_g}(L-x) \quad \text{—— (4)}$$

by integrating both side we get

$$\frac{dy}{dx} = \frac{W}{YI_g}\left[Lx - \frac{x^2}{2}\right] + C_1 \quad \text{—— (5)}$$

(C_1 = constant)

At A the beam is horizontal i.e $x=0$ $\frac{dy}{dx} = 0$

eq^n 5 ~~become~~ we have $C_1 = 0$

$$\frac{dy}{dx} = \frac{W}{YI_g}\left(Lx - \frac{x^2}{2}\right) \quad \text{—— (6)}$$

Again Integral eq^n (6) we get

$$y = \frac{W}{YI_g}\left[L\frac{x^2}{2} - \frac{x^3}{6}\right] + C_2 \quad \text{—— (7)} \quad (C_2 = \text{const})$$

at $x=0$ $y=0$

$\therefore C_2 = 0$

Then eqⁿ (7) become $\quad Y = \frac{W}{YI_g}\left[L\frac{x^2}{2} - \frac{x^3}{6}\right]$ ——— (8)

eqⁿ (8) is the expression for depression of beam at x from A.

But depression at B is obtained by using $x = L$

$$Y = \frac{W}{YI_g}\left[L\frac{L^2}{2} - \frac{L^3}{6}\right]$$

$$Y = \frac{W}{YI_g}\left[\frac{2L^3}{6}\right]$$

$$\boxed{Y = \frac{WL^3}{3YI_g}} \text{ ——— (9) (Required eqⁿ)}$$

for rectangle section $I_g = \frac{bd^3}{12}$

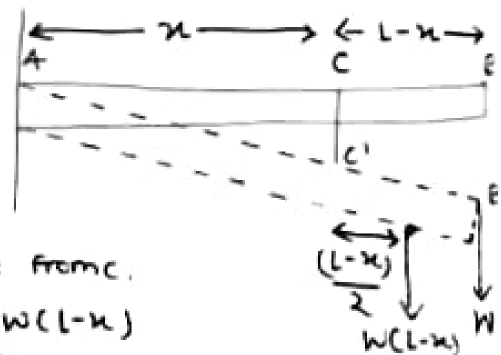
$$Y = \frac{WL^3}{3Y\frac{bd^3}{12}} = \frac{4WL^3}{Ybd^3}$$

for beam of circular crossection $I_g = \frac{\pi r^4}{4}$

$$Y = \frac{WL^3}{3Y\frac{\pi r^4}{4}} = \frac{4WL^3}{3Y\pi r^4}$$

(2) **Heavy Cantilever :-**

- Consider a cantilever AB fixed at end A & B end is loaded with weight W.
- let w be the beam per unit length.
- consider a section CB x distance from A & L is length of beam.
- CB has length $(L-x)$ & weight $w(L-x)$ is action on mid of CB at $\frac{(L-x)}{2}$ distance from C.



- so bending of beam at C due to W is $W(L-x)$ and due to w weigh of CB is given by

$$w(L-x)\frac{(L-x)}{2}$$
$$= \frac{w(L-x)^2}{2}$$

Total momentum due to external force

$$= W(L-x) + \frac{w}{2}(L-x)^2$$

for eqᵐ of beam the external couple is balaced by bending moment

$$\frac{YI_g}{R} = W(L-x) + \frac{w}{2}(L-x)^2 \text{ ——— (1)}$$

But $\frac{1}{R} = \frac{d^2y}{dx^2}$ $\quad (\because \frac{dy}{dx} << 1$ for small bending)

$$YI_g\frac{d^2y}{dx^2} = W(L-x) + \frac{w}{2}(L-x)^2$$
$$= W(L-x) + \frac{w}{2}\left[L^2 + x^2 - 2Lx\right] \text{ ——— (2)}$$

integrating eqⁿ (2) w.r.t. x we get

Integrating eqn (2) ~~we get~~ w.r.t x we get

$$YI_g \frac{dy}{dx} = w\left(lx - \frac{x^2}{2}\right) + \frac{w}{2}\left[l^2x - 2l\frac{x^2}{2} + \frac{x^3}{3}\right] + c_1$$

$$= w\left(lx - \frac{x^2}{2}\right) + \frac{w}{2}\left[l^2x - lx^2 + \frac{x^3}{3}\right] + c_1 \quad \text{--- (3)}$$

At $x = 0$, $\frac{dy}{dx} = 0$

$c_1 = 0$ we get it from eqn (3) & Integrating eqn (3) we get

$\cancel{YI_g \frac{dy}{dx} = w(lx - \frac{x^2}{2}}$ $YI_g y = w\left[l\frac{x^2}{2} - \frac{x^3}{6}\right] + \frac{w}{2}\left[l^2\frac{x^2}{2} - l\frac{x^3}{3} + \frac{x^4}{12}\right] + c_2$ —— (4)

const But $x = 0$ $y = 0$ $\therefore c_2 = 0$

$$YI_g y = w\left[\frac{lx^2}{2} - \frac{x^3}{6}\right] + \frac{w}{2}\left[\frac{l^2x^2}{2} - \frac{lx^3}{3} + \frac{x^4}{12}\right]$$

The depression at B is obtained by puting $x = L$

$$YI_g y = w\left[\frac{ll^2}{2} - \frac{l^3}{6}\right] + \frac{w}{2}\left[\frac{l^2l^2}{2} - \frac{ll^3}{3} + \frac{l^4}{12}\right] \text{ —— (5)}$$

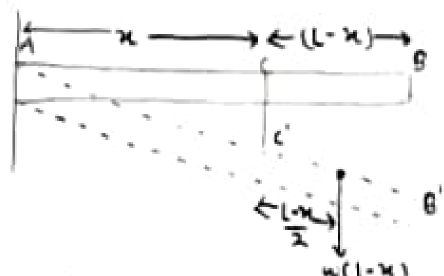
$$\Rightarrow YI_g y = \frac{wl^3}{3} + \frac{wl}{2}\cdot\frac{l^3}{4} = \frac{wl^3}{3} + \frac{w_1 l^3}{8}$$

$$\Rightarrow \boxed{y = \frac{l^3}{3YI_g}\left(w + \frac{3}{8}w_1\right)}$$ (where $w_1 = wl$ weight of beam) $\rightarrow$ (required eqn).

Comparing eqn $y = \frac{wL^3}{3YI_g}$ it is clear that w at free end increase $\frac{3}{8}$ of its own weight.

3) Cantilever loaded uniformly:-

Let's consider a cantilever AB of length l fixed at end A, while the other end B is free.



Let w be the load per unit length of the cantilever then we consider section CB which bends due to load $w(l-x)$ at distance $\frac{(l-x)}{2}$.

Then Bending moment $= w(l-x)\frac{(l-x)}{2} = \frac{w(l-x)^2}{2}$

$$\text{i.e. } \frac{YI_g}{R} = \frac{w(l-x)^2}{2} \quad \ldots\ldots (1)$$

But $\frac{1}{R} = \frac{d^2y}{dx^2}$ for small amount of bending

eqn (1) become $YI_g\frac{d^2y}{dx^2} = \frac{w(l-x)^2}{2}$ $\quad\ldots\ldots (2)$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{w}{2YI_g}(l-x)^2 \quad \ldots\ldots$$

by integrate eqn (2) we get.

$$\Rightarrow \frac{dy}{dx} = \frac{w}{2YI_g}\left(l^2x + \frac{x^3}{3} - \frac{2lx^2}{2}\right) + c_1 \quad (c_1 = \text{const})$$

using condition $x=0$, $\frac{dy}{dx}=0$ $c_1=0$

$$\frac{dy}{dx} = \frac{w}{2YI_g}\left[L^2x + \frac{x^3}{3} - Lx^2\right] \quad \text{——— (3)}$$

Integrating ~~eq~~^{3} w r t x we get

$$y = \frac{w}{2YI_g}\left[L^2\frac{x^2}{2} + \frac{x^4}{12} - \frac{Lx^3}{3}\right] + c_2$$

at $x=0$ $y=0$

$c_2=0$

$$\therefore y = \frac{w}{2YI_g}\left[\frac{L^2x^2}{2} + \frac{x^4}{12} - \frac{Lx^3}{3}\right] \quad \text{——— (4)}$$

So the depression at end B is obtained by taking $x=L$

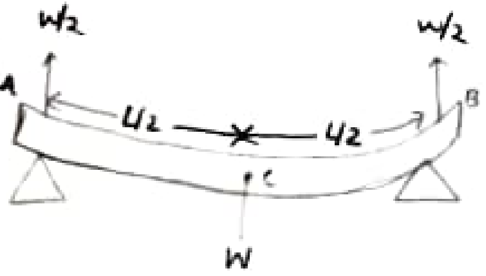
$$\Rightarrow y = \frac{w}{2YI_g}\left[\frac{L^2L^2}{2} + \frac{L^4}{12} - \frac{LL^3}{3}\right]$$

$$= \frac{w}{2YI_g}\left[\frac{L^4}{2} + \frac{L^4}{12} - \frac{L^4}{3}\right]$$

$$\Rightarrow y = \frac{wL}{2YI_g}\left(\frac{L^3}{4}\right)$$

$$\Rightarrow \boxed{y = \frac{WL^4}{8YI_g}}$$ (where $W = wL$) the total Load on beam.

(4) **Beam Supported at both end & Loaded at centre :-**



- Let us consider a beam 'AB' of length 'L' supported on ~~two~~ tow knife edges at both end & a weight 'W' is at it's centre. The upward rexn at each knife edge is $\frac{W}{2}$ as shown.

- The middle part of the beam such as at point C, it is horizontal so the beam is asumed to be made up of two contilever AC & BC of length $L/2$ fixed at C bending under load $W/2$.

so depression at C is

$$y_m = \frac{\left(\frac{W}{2}\right)\left(\frac{L}{2}\right)^3}{3YI_g} = \frac{WL^3}{38YI_g}$$

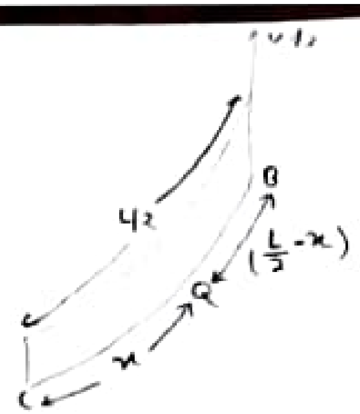
$$\cancel{y_m = \frac{WL^3}{38YI_g}}$$

Let us consider a section QB at distance x from from C as shown.

The external bending moment due to load $\frac{W}{2}$ about Q is $\frac{W}{2}\left(\frac{L}{2}-x\right)$

For equilibrium bending moment YI_g/R.

$$\frac{YI_g}{R} = \frac{W}{2}\left(\frac{L}{2}-x\right) \qquad (1)$$



For small bending $\frac{1}{R} = \frac{d^2y}{dx^2}$

Eqn (1) become $YI_g \frac{d^2y}{dx^2} = \frac{W}{2}\left(\frac{L}{2}-x\right)$

$$\frac{d^2y}{dx^2} = \frac{W}{2YI_g}\left(\frac{L}{2}-x\right) \qquad (2)$$

integrating equation (2) w.r.t x

$$\frac{dy}{dx} = \frac{W}{2YI_g}\left[\frac{Lx}{2} - \frac{x^2}{2}\right] + C_1$$

slope is zero i.e $\frac{dy}{dx} = 0 \therefore C_1 = 0$

then $\frac{dy}{dx} = \frac{W}{2YI_g}\left[\frac{Lx}{2} - \frac{x^2}{2}\right]$ ——— (3).

Again, integrating eq 3 w.r.t x weget

$$y = \frac{W}{2YI_g}\left[\frac{Lx^2}{4} - \frac{x^3}{6}\right] + C_2$$

at $x=0$ $y=0$

$C_2 = 0$

Then $y = \frac{W}{2YI_g}\left[\frac{Lx^2}{4} - \frac{x^3}{6}\right]$

depression produced at $x = \frac{L}{2}$ is obtained as

$$y = \frac{W}{2YI_g}\left[\frac{L}{4}\left(\frac{L}{2}\right)^2 - \frac{1}{6}\left(\frac{L}{2}\right)^3\right]$$

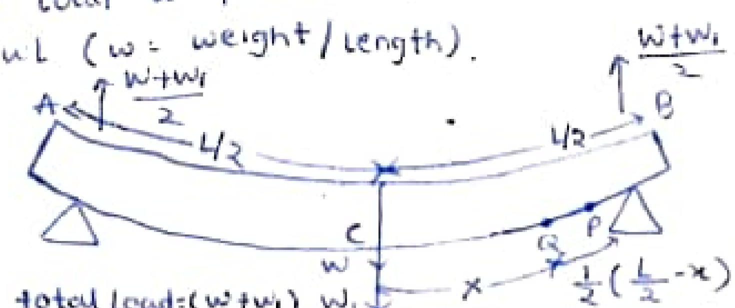
$$y = \frac{W}{2YI_g}\left[\frac{L^3}{16} - \frac{L^3}{48}\right]$$

$$= \frac{W}{2YI_g}\left[\frac{2L^3}{48}\right]$$

$$\boxed{y = \frac{WL^3}{48YI_g}}$$

(5) Beam is supported at two ends, loaded at it's centre :-

let use beam AB of length L supported at its two end A B wi[th] knife edge & load w placed at centre C. let W_1 be the weigh[t] of beam $W_1 = wL$ (w = weight / length).



- total load applied at C = external load (w) + weight of beam (W_1)
($w_1 = w + w_1$)
- The reaction at each knife edge in upward direction is $\frac{w+w_1}{2}$
- The beam is combination of two inverted cantilevers AC & BC of length L/2 & fixed point C is loaded with $\frac{w+w_1}{2}$ as shown in fig.
- If we consider GB of cantilever CB at a distance x from C & weight $w(\frac{L}{2}-x)$ acting at distance $\frac{1}{2}(\frac{L}{2}-x)$ at point P.

- The total external momentum at point G is sum of momentum of $\frac{w+w_1}{2}$ about G & moment of $w(\frac{L}{2}-x)$ about G.

$$= \frac{w+w_1}{2}\left(\frac{L}{2}-x\right) - w\left(\frac{L}{2}-x\right)\frac{1}{2}\left(\frac{L}{2}-x\right)$$

$$= \frac{w+w_1}{2}\left[\frac{L}{2}-x\right] - \frac{w}{2}\left(\frac{L}{2}-x\right)^2 \quad \text{——— (1)}$$

at eq^n condition. $\frac{YI_g}{R} = \frac{w+w_1}{2}\left[\frac{L}{2}-x\right] - \frac{w}{2}\left(\frac{L}{2}-x\right)^2$ ——— (2)

but we know for small amount bending $\left(\frac{dy}{dx}\right)^2$ is negligible

$$\therefore \frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\text{Eq}^n (2) \Rightarrow YI_g\frac{d^2y}{dx^2} = \frac{w+w_1}{2}\left[\frac{L}{2}-x\right] - \frac{w}{2}\left(\frac{L}{2}-x\right)^2$$

$$\Rightarrow YI_g\frac{d^2y}{dx^2} = \frac{w+w_1}{2}\left[\frac{L}{2}-x\right] - \frac{w}{2}\left[\frac{L^2}{4}+x^2-Lx\right] \quad \text{——— (3)}$$

Integrate eq^n 3 w.r.t x.

$$\Rightarrow YI_g\frac{dy}{dx} = \frac{w+w_1}{2}\left[\frac{Lx}{2}-\frac{x^2}{2}\right] - \frac{w}{2}\left[\frac{L^2x}{4}+\frac{x^3}{3}-\frac{Lx^2}{2}\right] + C_1 \quad \text{——— (4)}$$

at $x=0$, $\frac{dy}{dx}=0$

$C_1 = 0$

The amount of depression is obtained by integrating eq^n 4 $x=0$ to

$$\Rightarrow YI_g\, y = \frac{w+w_1}{2}\int_0^{L/2}\left[\frac{Lx}{2}-\frac{x^2}{2}\right]dx - \frac{w}{2}\int_0^{L/2}\left[\frac{L^2x}{4}+\frac{x^3}{3}-\frac{Lx^2}{2}\right]$$

$$\Rightarrow YI_g\, y = \frac{w+w_1}{2}\left[\frac{Lx^2}{4}-\frac{x^3}{6}\right]_0^{L/2} - \frac{w}{2}\left[\frac{L^2x^2}{8}+\frac{x^4}{12}-\frac{Lx^3}{6}\right]_0^{L/2}$$

$$\Rightarrow 2YI_g\, y = (w+w_1)\left[\frac{L}{4}\left(\frac{L}{2}\right)^2-\frac{1}{6}\left(\frac{L}{2}\right)^3\right] - \frac{w}{2}\left[\frac{L^2}{8}\left(\frac{L}{2}\right)^2+\frac{1}{12}\left(\frac{L}{2}\right)^4-\frac{L}{6}\left(\frac{L}{2}\right.\right.$$

$$= (w+w_1)\frac{L^3}{24} - wL\left(\frac{3L^3}{192}\right) = \frac{L^3}{24}\left[w+w_1-\frac{3w_1}{\cdot}\right]$$

$$= \frac{L^3}{24}\left[w+\frac{5}{8}w_1\right]$$

- when the weight of ... into account then it ... increased by $\frac{5}{8}$ times.

$$\boxed{y = \frac{L^3}{48\,YI_g}\left[w+\frac{5}{8}w_1\right]}$$

Reynolds Number is less than 2000, but for turbulent flow Reynolds No is above 3000. But when Reynolds Number lies between 2000 to 3000, then the flow is unstable it may change from streamline to turbulent or vice versa.

Poiseuille's formula for flow of liquid through capillary tube :

Poissuille derived an expression for the rate of flow i.e. volume of liquid flowing per second through a narrow horizontal tube of uniform bore under a constant pressure difference between its ends. The assumption used are as follows :

(i) The flow of liquid s streamlined, parallel to the axis of tube.

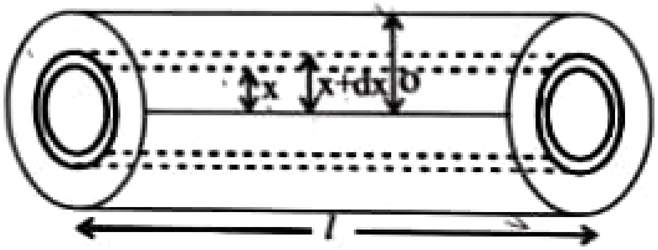
(ii) There is no Radial flow i.e., the pressure over any cross section is constant.

(iii) There is no acceleration of the liquid at any instant.

(iv) The liquid in contact with the walls of the tube is at Rest.

Figure shows a liquid flowing trough a narrow horizontal tube of radius r, length l under a constant pressure difference (P) applied between its ends. The velocity of liquid in the tube is maximum along axis and is zero at the walls of the tube.

Let us consider a coaxial cylindrical shell of the liquid of radius x and thickness dx. The liquid layer at distance x has velocity v and liquid layer at distance x + dx from the axis has velocity v - dv.



Hence force of viscosity on the cylindrical shell is given by Newton's formula

$$F = -\eta A \frac{dv}{dx}$$ where η is coefficient of viscosity of liquid.

$$\Rightarrow F = -\eta(2\pi x l)\frac{dv}{dx} \qquad \text{.......... (1)}$$

A = Surface Area of the shell is $2\pi x dx$.

The negative sign means that force of viscosity acts opposite to driving force.

The force which tends to drive the liquid in the shell is (pressure difference across its ends) × Area of cross section

$$\text{i.e., } F' = P(\pi x^2) \qquad \text{.......... (2)}$$

∴ For Steady flow of liquid these two forces must balance each other. So F = F'

$$\Rightarrow -\eta(2\pi x l)\frac{dv}{dx} = P(\pi x^2)$$

$$\Rightarrow \frac{dv}{dx} = \frac{-Px}{2\eta l}$$

$$\Rightarrow dv = \frac{-Px\,dx}{2\eta l} \qquad \text{.......... (3)}$$

Integrating equation (3) we get

$$\Rightarrow v = \frac{-p}{2\eta l}\left(\frac{x^2}{2}\right) + C \qquad \text{.......... (4)}$$

(where C is a constant of Integration)

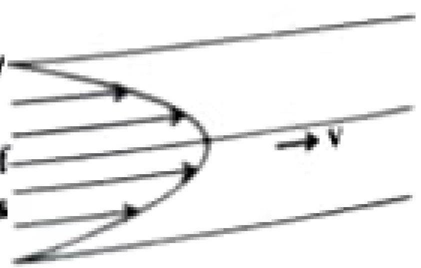
But when x = r (at the wall of the tube)

v = 0 (liquid is at Rest)

equation gives $0 = \frac{-pr^2}{4\eta l} + C \Rightarrow C = \frac{pr^2}{4\eta l}$ .... (5)

Using equation (5) in equations (4) we get :

$$v = \frac{P}{4\eta l}\left(r^2 - x^2\right) \qquad \text{.......... (6)}$$



This represents equation of parabola. So the profile of velocity distribution of liquid is parabola as shown.

The length of Arrows are proportional to the velocities of their respective positions. The velocity is maximum along the axis

($x = 0$) and zero at the walls of the tube ($x = r$).

Now the volume of the liquid flowing per second through the cylindrical shell of face area $2\pi x\,dx$ is given by

$$dV = \text{(velocity) Face Area of shell}$$
$$= (v)(2\pi x\,dx)$$

$$\text{or } dV = \frac{p}{4\eta l}\left(r^2 - x^2\right)2\pi x\,dx$$

$$\Rightarrow dV = \frac{p}{2\eta l}\left(r^2 - x^2\right)\pi x\,dx$$

Hence the volume of the liquid V flowing per second through the whole tube is obtained by integrating the above expression between limits $x = 0$ and $x = r$.

$$\text{i.e., } V = \frac{\pi p}{2\eta l}\int_0^r \left(r^2 - x^2\right)x\,dx = \frac{\pi p}{2\eta l}\left[r^2\frac{x^2}{2} - \frac{x^4}{4}\right]_0^r$$

$$\text{or } V = \frac{\pi p}{2\eta l}\left[\frac{r^4}{2} - \frac{r^4}{4}\right]$$

$$\Rightarrow \boxed{V = \frac{\pi p r^4}{8\eta l}} \qquad \ldots\ldots\ldots\ldots$$

which is poiseuille's formula.

Limitations and corrections :

In deriving poiseuille's formula two important aspects have been neglected. It was assumed that pressure difference across two ends of pipe was used up against viscosity, but actually a part of this pressure difference is used up in imparting KE to the liquid. Secondly it was assumed that liquid flowing in the tube had no acceleration and is streamline as it enters the tube. But in fact liquid while entering the tube is accelerated.

(a) Kinetic Energy correction : Let P' be the ...

... resistance of liquid and imparting KE to the

$P'V + V^2\rho/\pi^2 r^4$

$$\therefore PV = P'V + \frac{V^2\rho}{\pi^2 r^4} \Rightarrow \boxed{P' = P - \frac{V^2\rho}{\pi^2 r^4}}.$$

(b) **Correction for Acceleration :** The motion of liquid as it enters the tube is acceleraated and does not become streamline until after covering a large distance.

So error is eliminated if the effective length of the tube is taken to be $(l + 1.64r)$ where r is radius of tube.

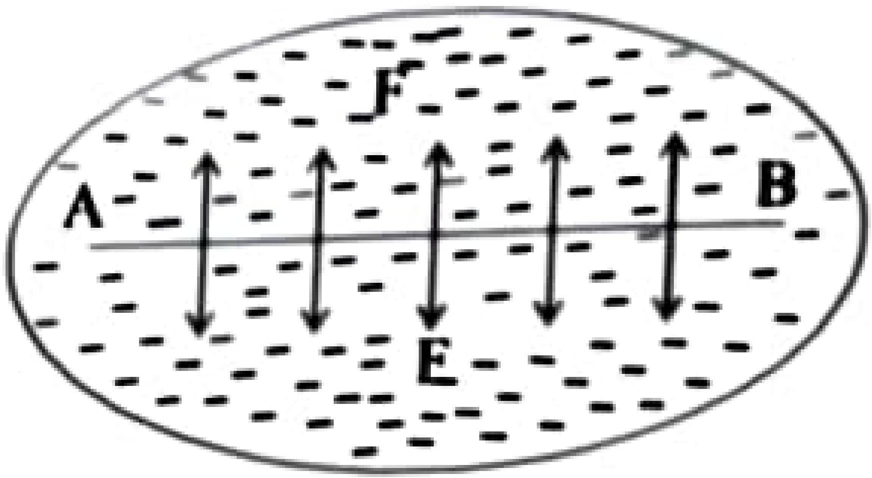
So the consideration of Two corrections leads to modified form of poiseuille's formula as

$$= \boxed{V = \frac{\pi\left(P - \rho V^2/\pi^2 r^4\right)}{8\eta(l + 1.64r)}}$$

SURFACE TENSION

The property of a liquid by virtue of which the surface of a liquid behaves like a stretched membrance and has the tendency of contracting is known as surface tension. As a result the surface of liquid can support heavier objects over it. For example a needle can be placed carefully on water surface so that it floats without sinking. Some insects also walk over liquid surface with little depressions in the regionof contact. Due to the property of surface tension the liquid surface tends to contract i.e. it tends to make the minimum surface area. But for a given volume, the surface area of a sphere is minimum. It is for this reason that the shape of mercury drop or rain drop or water drop are spherical (neglecting the effect of Gravity). The property of surface tension of liquid can be explained on the basis of molecular theory of matter.

If we imagine a line AB of length l in the free surface of a liquid at rest then the force of surface tension is defined as the force acting per unit length on either side of the imaginary line. This force acts in a direct[illegible]dicular to the line and is tangential to the liquid surface.



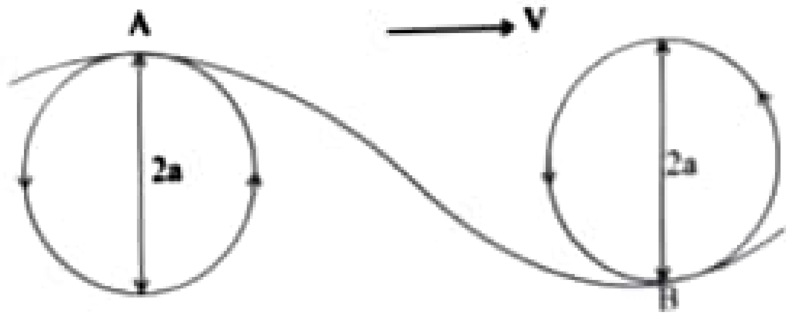
So if F is the force acting on either side of the line AB of length l, then surface ten

$$\boxed{T = \frac{F}{l}}$$

In SI system surface tension (T) is expressed in newton/meter (Nm^{-1}). In CGS sys
expressed in dyne/cm. The dimensional formula of surface tension is $[ML^0T^{-2}]$.

Gravity Waves

A wave is a type of disturbance which travels through a medium and is due to the transfer of motion from particle to particle of the medium. But there is no material transference of the medium. The wavelength of waves formed on the surface of sea is very large and such waves are called gravity waves. In these types of waves the curvature of the surface is small and gravity dominates over surface tension. So in this type of wave there is transverse vibration i.e. surface moves up and down in a periodic manner. In addition there is periodic horizontal motion. It is assumed that the drops of liquid near the surface describe circular path in anticlockwise direction in the vertical plane and waves propagate in the direction from left to right in the form of crests and troughs.



If V is the velocity of the waves along the horizontal direction and a is radius of the circle and T_0 is the time period of motion of drop (it is also the time taken by the waves to cover distance equal to wavelength λ), then if V_1 is the velocity of particle on crest A.

We have $V_1 = V - \frac{2\pi a}{T_0}$ .... (1)

[as velocity = Angular velocity × amplitude = ωa = ωa = $\frac{2\pi a}{T_0}$]

But at the through B, the velocity of the particle is given by

$V_2 = V + \frac{2\pi a}{T_0}$ .... (2)

The pressure at the liquid surface is equal to atmospheric pressure everywhere

i.e $P_A = P_B = P_0$ (Atmospheric pressure)

Applying Bernoulli's theorem at A and B

$$P_0 + \frac{1}{2}\rho V_1^2 + \rho gh = P_0 + \frac{1}{2}\rho V_2^2$$

or $V_1^2 + 2gh = V_2^2$ .... (3)

Now usign the values of V_1 and V_2 form eqⁿ (1) & (2) in eqⁿ (3) we get :

$$\left(V - \frac{2\pi a}{T_0}\right)^2 + 2gh = \left(V + \frac{2\pi a}{T_0}\right)^2$$

$$\text{i.e} \left(V + \frac{2\pi a}{T_o} \right)^2 - \left(V - \frac{2\pi a}{T_o} \right)^2 = 2gh$$

$$\text{i.e. } (4V)\left(\frac{2a\pi}{T_o} \right) = 2gh \text{ i.e } \frac{4\pi a V}{T_o} = gh \qquad \text{.... (4)}$$

But $h = 2a$, $T_o = \lambda / v$

So eq^n becomes

$$\frac{4\pi a V}{\lambda / v} = (g)(2a) \Rightarrow V^2 = \frac{g\lambda}{2\lambda} \text{ or } V = \sqrt{\frac{g\lambda}{2A}} \qquad \text{......(5)}$$

Which is the expression for velocity of grevity waves.

Effect of surface Tension on Gravity waves (Ripples)

The wavelength of waves formed on the surface of ponds and lakes is samll and such small waves are called ripples. In these types of waves surface Tension plays a very important role due to the face that in case of a curved liquid surface there exists pressure difference due to surface Tension is an excess pressure exists across the curved liquid surface directed from concave side to the convex side and is given by $P = T\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$ Where T is surface Tension of liquid R_1 & R_2 are radii of curvatures.

The equation of simple harmonic wave traveling over a liquid is given by

$$y = r\sin\left(\frac{2\pi x}{\lambda}+b\right) \qquad \text{..... (1)}$$

Where r is amplitude, λ is wavelength, y is displacement of a point along vertical or y axis is the abscissa of the point from some origin.

b is some constant.

Then $$\frac{dy}{dx} = \frac{2\pi r}{\lambda}\ \text{Cos}\left(\frac{2\pi x}{\lambda}+b\right)$$

$$\frac{d^2y}{dx^2} = -\left(\frac{2\pi r}{\lambda}\right)\left(\frac{2\pi}{\lambda}\right)\ \text{Sin}\left(\frac{2\pi x}{\lambda}+b\right)$$

or, $$\frac{d^2y}{dx^2} = \frac{-4\pi^2 r}{\lambda^2}\ \text{Sin}\left(\frac{2\pi x}{\lambda}+b\right) \qquad \text{..... (2)}$$

As the wave surface is cylindrical one of the radii is infinite then putting $R_2 = \infty$, $R_1 = R$

We get $P = T\left(\frac{1}{R} + \frac{1}{\infty}\right)$ or $P = \frac{T}{R}$ ...... (3)

But $\frac{1}{R} = \frac{d^2y/dx^2}{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}}$

or $\frac{1}{R} = \frac{d^2y}{dx^2}$ as $\frac{dy}{dx}$ is small in comparision to 1.

At point A, slope is zero i.e $\frac{dy}{dx} = 0 \Rightarrow \text{Cos}\left(\frac{2\pi x}{\lambda} + b\right) = 0$

then $\text{Sin}\left(\frac{2\pi x}{\lambda} + b\right) = \pm 1$

Using Maximum value of $\text{Sin}\left(\frac{2\pi x}{\lambda} + b\right) = +1$ as $\frac{d^2y}{dx^2} < 0$ at A

we get $\frac{d^2y}{dx^2} = \frac{-4\pi^2 r}{\lambda}$ or $\frac{1}{R} = \frac{-4\pi^2 r}{\lambda^2}$ --- (4)

Hence at point A, excess pressure along CA due to surace Tension is $\frac{T}{R}$ or along AC $-\frac{T}{R} = -T\left(\frac{-4\pi^2 r}{\lambda^2}\right)$ or $\frac{4\pi^2 rT}{\lambda^2}$. As a result the excess force on an element of area da of thic unity is $\frac{4\pi^2 rT}{\lambda^2}$ da along AC in downward direction.

Also the weight of the liquid column AA'C'C is mg = Volume × dencity × g = (da) y where ρ is density of liquid.

As a result the total downward force is

$y_A \rho dag + \frac{4\pi^2 rT}{\lambda^2} da = \rho y_A da\left(g + \frac{4\pi^2 T}{\lambda^2}\right)$

---- (5) as $y_A = r$ the ampl

From the above discussion it is found that the effect of surfa...

g to $g + \frac{4\pi^2 T}{\rho\lambda^2}$

Then as we know velocity of gravity waves is $V = \sqrt{\frac{g\lambda}{2\pi}}$, In this discussion we get the expression for velocity of waves controlled by gravity and surface Tension as $V = \sqrt{\frac{\lambda}{2\pi}\left(g + \frac{4\pi^2 T}{\rho\lambda^2}\right)}$ .....(6)

So if λ is verysmall as in case of small or short waves known as Ripples, the second term in eqⁿ (6) dominates over the first term

so that $V = \sqrt{\frac{\lambda}{2\pi} \cdot \frac{4\pi^2 T}{\rho\lambda^2}}$ i.e $V = \sqrt{\frac{2\pi T}{\rho\lambda}}$ ..... (7)

Which is controlled by surface Tension.